Mathematics and Islam: Background Information

There are a number of developments in Mathematics that originate from Islamic scholarship during the early period of Islam. Some of the ideas were certainly not discovered by Islamic scholars but they were the first to successfully disseminate the information beyond their own lands and throughout the known world.

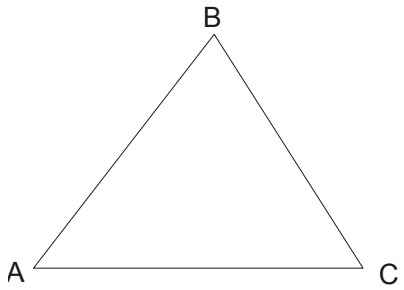
The high period of the Islamic empire was approximately 750 – 1200 AD. Euclid of Alexandria (ca. 325 BC–265 BC) was one of the first to formalise geometrical theories and is traditionally known as the father of geometry.

Triangles and the Umbra Recta

The understanding of trigonometry that the Islamic community had developed predominantly facilitated the design and construction of some of the great buildings of the Islamic Civilisation. The use of trigonometry also enabled the influenced the development of astronomy and navigation.

Although trigonometry is only briefly discussed in the current Mathematics component of the National Curriculum, there is a very important link between the use of trigonometry that Islamic mathematicians used and the similar triangle.

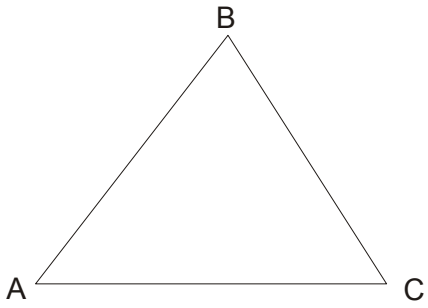
Briefly: If the lengths of two sides of a triangle are known and the angle between or the angles of two apexes and the length of one side in between is known the exact shape and size of a triangle can be determined.



e.g. sides AB, AC and angle BÂC

or angles BÂC, BĈA and side AC

For similar triangles only the shape is known and not the exact size; any two angles are known.



e.g. angles AB̂C, BĈA

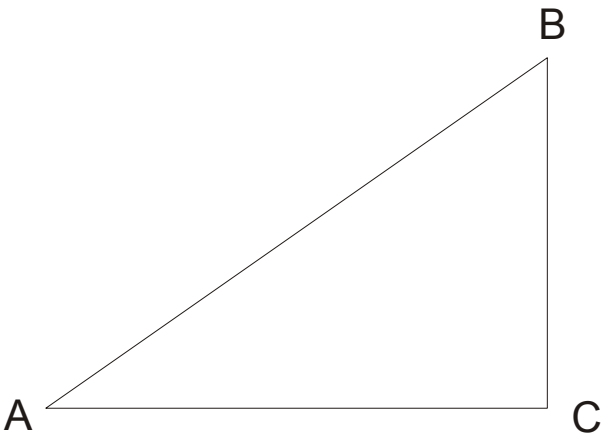
or BĈA, CÂB

or CÂB, AB̂C

By using similar triangles students will be able to recreate the use of some of the geometrical instruments that the Islamic mathematicians used to calculate heights during surveying and measure latitude in navigation.

The Astrolabe was a tool for making such measurements and would be engraved with an *Umbra Recta* or 'shadow square'. This was constructed on the basis of similar triangles and trigonometry.
The relationship between the base line and the height of an object is the tangent of the angle.

$$\text{Tangent (tan)} = \frac{\text{opposite}}{\text{adjacent}}$$



$$\tan C\hat{A}B = \frac{BC}{AC}$$

By knowing the base line and the angle CÂB the height BC can be calculated. The Umbra Recta is simply a geometrical trigonometry calculator.

Arabic numerals

Zero

Zero is one of the most commonly used concepts in mathematics, but it is difficult to grasp the fundamental importance of having a figure to represent it. Islamic scholars did not invent the concept of zero, but what they did do was present to the Western World the idea of place value. Until this point, number was represented by letters from the Roman alphabet. I, II, III etc. However, there was no symbol for zero and when large numbers were represented the length of the string of symbols became unwieldy.

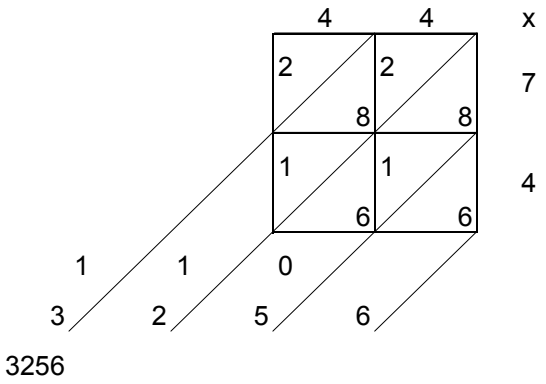
Place value and ten digits reduced the problem to a shorter representation of number and made mechanical arithmetic much simper and faster.

Multiplication

The Arabic numerals aid greatly in the use of more complicated calculations. Methods of mechanically carrying out long multiplication would not work without place value.

$$\begin{array}{r} 123 \\ \times \quad 45 \\ \hline 4920 \\ + \quad 615 \\ \hline 5535 \end{array}$$

The Arabic mathematicians used a different method, which shows exactly how place value fits in.



By knowing the tables up to 9x9 the answer to each small multiplication is placed into the grid with the upper triangle for the tens and the lower for the unit. When totalling the whole, the digits are added along the diagonal lines and the answer is formed.

Lesson: 1 of 4

Similar Triangles and their properties

Duration: 30 min

Learning Opportunities:
Investigating triangles: Are individual triangles similar, congruent or neither?
Solving simple puzzles such as finding out the unknown lengths in pairs of similar triangles.

Learning Outcomes:
Pupils should be able to describe similar triangles.
Pupils should be able to identify similar triangles.
Pupils should be able to use simple scaling factors between different triangles.

Plenary:
Sorting triangles into categories: Provide sets of 10-20 triangles. Pupils sort into their self defined categories, then into equilateral, isosceles, scalene and right-angled triangles.

Main Lesson Content:

- Use the PowerPoint presentation to introduce properties of triangles.
- Pupils can construct different triangles using compass and rule and protractor.
- Drawing round pre-made templates may be suitable for lower ability students.
- Provide list of parameters to ensure the triangles constructed will be congruent, similar and neither.

Use the idea of similar triangles to solve problems. If there is time the pupils could identify the triangles, construct them and measure the unknown side to compare this with their calculated solution.

Summary:
Draw together the key ideas, similar triangles, solving problems and introduce the concept that these methods were used for practical purposes such as surveying, engineering and construction of buildings.

Lesson: 2 of 4

Using the Umbra Recta

Duration: 30 min

Learning Opportunities:
Visit to the MUSEUM OF THE HISTORY OF SCIENCE to see the astrolabe and other historical measuring instruments.
Making an *Umbra Rectra*, a copy of one in the museum, and use it to measure the height of an unknown building.
Making a quadrant and using a measured angle to find unknown heights or distances.

Learning Outcomes:
Understanding of how similar triangles can be used to find unknown distances.
Understanding of how to use the umbra recta or quadrant to make measurements.

Plenary:
Perhaps investigate the length of shadows produced by a lamp at different heights above a gnomon (a pencil or a straw). This will give a visual idea that different angles produce different lengths of shadow. Perhaps read short extract form A. Conan- Doyle's "The Musgrave Ritual".

Main Lesson Content:
- Cut out and make the umbra recta, which can be seen on the back of the astrolabe at the museum.
- The *Umbra recta* can be used in conjunction with the instructions in Chaucer's work (1319).
- Pupils can measure and record heights of buildings and features around the museum or school grounds.
- For the more able students, the idea of the tangent trig functions could be introduced by using the protractor in conjunction with the Umbra Recta.

Summary:
Discuss the ease or difficulties the pupils had in measuring heights with their *Umbra Recta* and how we would do the same measurements today.

Lesson: 3 of 4

Zero and place value

Duration: 30 min

Learning Opportunities:
Investigating how useful zero is in everyday arithmetic.
Investigating how place-value has superseded Roman Numerals and simplified numbers.

Learning Outcomes:
Pupils will know that the Islamic world introduced zero to the "West".
Pupils should be able to see the usefulness of being able to describe a zero quantity.
Pupils should be able to see the benefit of place value in simple arithmetic.

Plenary:
Give the pupils the task of counting a number of different objects and devising some way of recording how many items there are of different types, with out using any of the digits 0 – 9. (Coloured counters, cards or Lego bricks for example.) Pupils may use tally charts of pictorial diagrams, for example, or may simple substitute different words or symbols for the Arabic numbers. E.g. If they have four red bricks, they may simple denote the number '4' with the word red so when they have 4 circular discs they will say the have "red" number of discs.

Main Lesson Content:
- Using the Excel spread sheet provided or other method (digits on cards) the pupils should try writing numbers in both Roman and Arabic numbers.
- They can explore how to carry out addition and subtraction in Roman Numerals and compare this to the way they would do the same arithmetic in conventional Arabic Numbers.

Summary:
For the more able students the ideas of using bases other than base 10 could be introduced, such as binary and hexadecimal systems. Weaker students could develop a set of 10 symbols to replace 0 – 9 which pictorially represent the number (similar to the Roman five "V" that represents the shape of a hand with four fingers together and the thumb to form the V).

Lesson: 4 of 4

Long multiplication

Duration: 30 min

Previous Knowledge:
Use of place value and the difference between using Roman and Arabic numbers.

Learning Opportunities:
Investigating how traditional long multiplication and an Islamic influenced method differ.
Evaluation of methods, which is easier to use? Which helps explain the multiplication method and significance of place-value?

Learning Outcomes:
Pupils should be able to use two methods of long multiplication.

Plenary:
A long multiplication calculation should be set. The pupils can calculate the answer by the method they are most familiar. Then see if they can explain to a partner how they carried out the process.

Main Lesson Content:

- The Arabic method can be taught and blank grids or the excel spreadsheet used to carry out further calculations.
- Students can evaluate the ease or difficulty of using this method over the more traditional Western Method.

The more able pupil may like to investigate the link between multiplication and division, using only products within the tables up to 12 x 12

e.g. 11 x 9 = 99

& 11 x X = 99

Here the x is representative of the unknown number 9. We could also write this as

$$99 \div 11 = ?$$

This demonstrates the beginnings of using algebra, another Arabic concept.

Summary:
Discuss the importance of the various Arabic influences there have been on our mathematical knowledge in the West and how we have benefited from them.