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John Dee's *Tyrocinium Mathematicum*: new evidence for a lost text

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Edinburgh University Library houses a collection of mathematical books bequeathed in 1635 by James Douglas, secretary to James VI. Many came from the library of the Scottish mathematician and royal physician John Craig.¹ Shelfmark Dh.5.195 forms part of Douglas's bequest and is a copy of Francesco Maurolyco, *Opuscula mathematica* (Venice, 1575). Bound in as a leaf preceding the title page is a manuscript letter in a neat italic hand from one Gratianus Niger to John Craig, in response to Craig's request for specific information from a now lost mathematical work by Dee.

Text²

Perspecta iam satis luculenter singulari vestra humanitate cum erga me, tum erga vnicum praeceptorem meum, Nobilem virum, Dominum Joannem Dee Brytannum, Lubenter polliceor, & hoc scripto testatum relinquo hanc ita initam amicitiam, nobis vtrisque gratissimam fore. *Atque* en tibi interim (Doctissime Craige) praeceptoris mei (quod requiris), primi libri, Theorema 12^m; ex eo volumine, cui (ante 18 annos absoluto) titulum praefixit, *Tyrocinium Mathematicum*

Theor. 12^m

Solidum quod fit ex semiperimetro alicuius Trianguli [word erased] rectilinei, in semidiametrum Circuli eidem inscriptibilis, et ex eo quod inde procreatur, in eandem semidiametrum: Aequale est illi Solido quod fit ex tribus illis residuis, quae ex singulorum laterum a Semiperimetro eiusdem Trianguli subtractione fiunt.

Amicus vester, Gratianus Niger
A^o 1578. Decemb. 18
Francofurti iuxta Oderam

Translation

Your singular humanity both towards me, and towards my especial teacher, that noble Briton, Master John Dee, is already splendidly evident enough. I willingly offer this letter as witness, so that the friendship we have already begun will be richly rewarding to each of us. And meanwhile see for yourself (most learned Craig) that which you seek: theorem 12 of the first book of my teacher's volume entitled *Tyrocinium Mathematicum*, which he completed 18 years ago.

Theorem 12

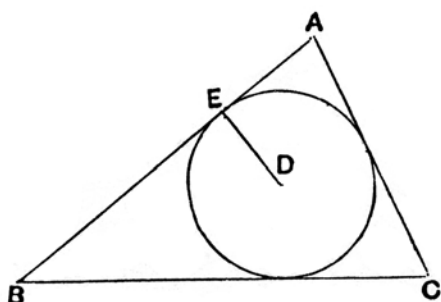
The solid which is made from the product of the semiperimeter of any rightlined triangle and the radius of the circle inscribable within it, together with the same radius, is equal to the solid which is made from the three residues left by subtracting each side from the semiperimeter of the same triangle.

Your friend, Gratianus Niger, 18
December 1578, at Frankfurt an der
Oder

Commentary

Gratianus Niger, the letter's author, has not so far been traced.³ However, the recipient John Craig is well-known: he had matriculated at Frankfurt an der Oder in 1573 and became professor of mathematics and logic there before graduating MD from the University of Basel in 1580 and then returning to Scotland in about 1582 to pursue a successful medical career.⁴

The date of the letter is significant. John Dee arrived in Frankfurt an der Oder on 11 December 1578 while on his 100 day journey “undertaken and performed to consult with the learned physitions and philosophers beyond the seas for her Majesties health recovering and preserving”.⁵ Dee had evidently taken part in mathematical as well as medical discussions while in Frankfurt.



Dee's theorem is about volumes constructed from dimensions in a plane figure. By Euclid IV, 4 a circle can be inscribed in any triangle ABC. Let the triangle's semiperimeter $\frac{1}{2}(AB + BC + CA)$ be S and the radius DE of the inscribed circle be R . Dee's theorem states

$$SR^2 = (S - AB)(S - BC)(S - CA)$$

Niger's letter provides no further evidence to reconstruct the mathematical context for this result. But there can be no doubt that it is closely related to Hero's theorem, which expresses the area of the triangle as $\sqrt{S(S - AB)(S - BC)(S - CA)}$.⁶ Dee's proposition can be readily derived from Hero's by dividing the triangle into its component parts ABD, BCD and CAD, and expressing each of these areas in terms of their longest side and the circle's radius. Dee's interest in turning a rule of measurement into solid geometry matches the focus of his annotations to Henry Billingsley's English Euclid of 1570, which begin in Book X and continue through the books of solid geometry. While Dee's mathematical style was not orthodoxly Euclidean, it may also be significant that his theorem preserves dimensional propriety by equating two solids, whereas Hero's proof required “ungeometrical” operations such as the product of two areas.⁷

In addition to the enunciation of the theorem, the letter also tells us that it came from a multi-book text called *Tyrocinium Mathematicum* (*TM*), dating from about 1560. No work of this title survives and it is not listed in Dee's *Compendious Rehearsall* (1592). But it is mentioned elsewhere, in an extended comment to the 1570 Euclid which Dee must have either suggested or approved. The comment appears deep in the theory of irrationals at Book X, 53, and follows a corollary translated from the edition of François Foix de Candale (*Flussas*). It is accompanied by a marginal note “M. Dee his booke called Tyrocinium Mathematicum”:

Although I here note vnto you this Corollary out of *Flussas*, yet, in very conscience and of gratefull minde, I am enforced to certifie you, that, many yeares, before the trauailes of *Flussas* (upon *Euclides* Geometricall Elementes) were published, the order how to deuide, not onely the 6 Binomiall lines into their names, but also to adde to the 6 Residuals their due partes: and farthermore to deuide all the other irrational lines (of this tenth booke) into the partes distinct, of which they are composed: with many other straunge conclusions Mathematicall, to the better vnderstanding of this tenth booke and other Mathematicall bookes, most necesssary, were by *M. Iohn Dee* inuented and demonstrated: as in his booke, whose title is *Tyrocinium Mathematicum* (dedicated to *Petrus Nonnius*, An. 1559) may at large appeare. Where also is one new arte, with sundry particular pointes, whereby the Mathematicall Sciences, greatly may be enriched. Which his booke, I hope, God will one day allowe him opportunitie to

publishe: with diuers other his Mathematicall and Metaphysicall labours and inuentions (f. 268r-v).

Billingsley had apparently studied *TM* and he here makes clear that it deals not only with irrationals but opens up a whole new art. This was just the direction in which Dee's Euclidean additions were aimed: "my desire is somewhat to furnish you, toward a more general art Mathematical then Euclides Elements, (remaying in the termes in which they are written) can sufficiently helpe you vnto" (f. 371r). The connection between *TM* and Dee's Euclidean contribution also exists on a pedagogical level. As a work evidently aimed at tyros, *TM* would have matched the tenor of many of Dee's additions to the *Elements*, which are often explicitly directed to young and tender students.⁸

There is also a more concrete link between the content of *TM* and Dee's Euclidean annotations. The theorem excerpted in 1578 by Gratianus Niger shows that *TM* included matters of solid geometry. It most likely covered other material which Dee added to the later books of Euclid, such as the "Theoremes and Problemes (whose vse is manifolde, in Spheres, Cones, Cylinders, and other solides)" occupying a sequence of 17 pages at the end of Book XII (ff. 381v-389v).

Many modern readers of the tantalising but frustrating lists of "unprinted Bookes and Treatises, ... some, perfectly finished: and some, yet unfinished" which Dee compiled in the 1590s must have wondered just how substantial these lost works really were. Billingsley's printed notice and Gratianus Niger's ability to quote a specific, identified theorem suggests that, at least in the case of *TM*, Dee's text was sufficiently complete to be shared and discussed. Moreover, it seems at least possible that Dee's notes on books X-XIII of Euclid represent further, perhaps reworked, material from this lost work.

If this interpretation of the likely character and content of *TM* is correct, it opens up some further questions. For instance, why did Dee compose an introductory work to accompany and elucidate the later stages of Euclid's *Elements* in about 1559 or 1560? As it happens, the date of composition of *TM* fits precisely into Dee's contemporary concerns and responsibilities: in about 1559 he took over the mathematical tuition of the thirteen-year-old Thomas Digges, giving him a strong reason for turning to pedagogy.⁹

More generally, does this reading of the relationship between *TM* and the 1570 Euclid tell us anything new about Dee's participation in that publication? Dee's famous 'Mathematicall Praeface' is so familiar that we are apt to assume it has a natural or inevitable place before Henry Billingsley's translation. By contrast, Dee's additions to the text of the *Elements* have scarcely been considered.¹⁰ But this judgement of relative significance may reverse their chronological and causal sequence. Billingsley's translation was a major undertaking and his collection of material from ancient and modern commentators must have been a long-term project. Long before the preface was considered, Dee would have been interesting for what he could contribute to the text, with *TM* a useful source for Billingsley to mine.

It is certainly striking that Dee's additions to Euclid show little sign of the hurried composition so characteristic of the 'Praeface'. The corollaries, problems and notes are methodically organised. Dee does date his note to Book XIII, 2 on 18 December 1569, which shows that he did not simply hand over a long-established text to Billingsley. But even this was leisurely compared to his deadlines in writing the 'Praeface'. His 'Groundplat' is dated 3 February 1570 and was printed on the 25th, while the preface was completed on the 9th. Dee refers several times to being "pinched with straightnes of tyme": "the Printer, hath looked for this Praeface, a day or two" and "still the Printer awayting, for my pen staying".¹¹ Was the 'Praeface' an afterthought, added only once Dee was already involved for his contributions to Books X-XIII? In that case, did Dee only come to write the 'Praeface' because he had earlier composed the *Tyrocinium Mathematicum* as tutor to Thomas Digges?

If this seems almost too neat a conclusion there is, as so often with Dee, at least one loose end. While *TM* is not mentioned in either the *Compendious Rehearsall* (1592) or the *Discourse Apologeticall* (1594/5), the latter lists another lost work: “De Triangulorum rectilineorum Areis – libri – 3 – demonstrati: ad excellentissimum Mathematicum Petrum Nonium conscripti – Anno – 1560”.¹² *TM* was dedicated to Nunez and that fact, combined with the coincidence of date, might suggest that this work on triangles is the same as *TM*. But while the link to Hero’s theorem makes it at least conceivable that Niger’s excerpt could have featured in a work on the area of triangles, the latter seems an unlikely home for material on irrational magnitudes. Neither does the treatise on triangles appear to promise the enticingly ambitious “new arte” which Dee trumpeted as part of *TM* in 1570. The *Compendious Rehearsall* has a briefer listing of the same book: “De triangulorum areis libri demonstrati 3. – A. 1560”. In his increasingly desperate attempts to demonstrate the breadth and number of his studies in the 1590s, did Dee simply embroider or confuse aspects of his earlier work?

I can’t attend the conference but would be grateful to receive any comments, criticisms or additional points. I can be contacted at stephen.johnston@mhs.ox.ac.uk.

¹ Douglas’s benefaction is noted in Jean R. Guild and Alexander Law (eds), *Edinburgh University Library 1580-1980: a Collection of Historical Essays* (Edinburgh, 1982), p. 48. The list drawn up in 1635 is at EUL Da.1.29/7.

² Expanded contractions are indicated by italics. My thanks to Edinburgh University Library for permission to reproduce the text of the letter.

³ Ernst Friedlaender (ed.), *Aeltere Universitäts-Matrikeln. I., Universität Frankfurt a.O.*, 3 vols (Leipzig, 1887-1891) includes two 16th-century individuals called Georgius Niger, one an Italian. There are also many entries under Schwarz, but none that seem to match with a Gratianus.

⁴ John Henry, ‘Craig, John (d. 1620?)’, *Oxford Dictionary of National Biography*, Oxford University Press, Sept 2004; online edn, Jan 2007.

⁵ James Orchard Halliwell, *The Private Diary of Dr John Dee, Camden Society* (1842), p. 5 and Dee’s *Compendious Rehearsall* in James Crossley (ed.), *Autobiographical Tracts of Dr. John Dee*, Chetham Miscellanies, vol. 1 (1851), p. 22.

⁶ On Hero’s theorem, see T.L Heath, *Euclid: the Thirteen Books of the Elements*, 3 vols (New York, 1956), II, 87-8 and M. Clagett, *Archimedes in the Middle Ages, I: The Arabo-Latin Tradition* (Madison, 1964), Appendix IV.

⁷ On Dee’s style, see Wayne Shumaker and J.L. Heilbron, *John Dee on Astronomy: Propaedeumata Aphoristica, 1558 and 1568* (Berkeley, 1978), p. 27.

⁸ See for example Euclid (1570), f. 380r: “*M. Dee* his devise, to helpe the imagination to young students in Geometry.” ‘Tirocinium’ originally referred to first military service or campaign or to young troops, but it broadened to mean the first experience or training in anything, and thus apprenticeship or pupilage. It may also be worth noting here that, despite the coincidence of titles, there was no connection between Dee’s work and that of the Scottish mathematician George Sinclair: *Tyrocinia mathematica, sive juniorum ad matheses addiscendas introductio, in quatuor tractatus, videlicet, arithmeticum, sphaericum, geographicum, & echometricum, divisa. Pro studiosa juventute ex Academia Glasguana cum laurea magisteriali hoc anno emittenda* (Glasgow, 1661). Certainly, nothing similar to the statement of Dee’s theorem I, 12 appears appropriately in place.

⁹ Stephen Johnston, “Like father, like son? John Dee, Thomas Digges and the identity of the mathematician”, in Stephen Clucas (ed.), *John Dee: Interdisciplinary Studies in English Renaissance Thought*, (Dordrecht, 2006), 65-84, p. 67.

¹⁰ Heilbron’s essay in *John Dee on Astronomy* is probably the most notable exception. It works through some of Dee’s problems and (at p. 22 n.74) corrects and expands the list of Dee’s contributions given in R. C. Archibald, “The First Translation of Euclid’s Elements into English and its Source”, *The American Mathematical Monthly*, 57 (1950), 443-452, p. 444.

¹¹ Euclid (1570), sigs. A.iiijv, c.iiijv and d.iiijv. Note that Dee does refer once to his lack of leisure in the later additions (f. 362r).

¹² John Dee, *Discourse Apologeticall* (1604), sig. B[1]r, no. 33.