Properties of Triangles 1

• Congruency
  – Two triangles that are exactly identical are known as congruent.
  – Test for congruency: can the two triangles be placed one on top of the other such that they exactly over lap?
Which of these triangles are congruent?
Are a and b congruent?
Are $a$ and $c$ congruent?
Are a and d congruent?
Congruent Triangles
Properties of Triangles 2

• Similar Triangles
  – Two triangles that have two angles the same size are known as similar.
    • Because the angles in a triangle always add to 180° then the third angle will also be the same.
  – Test for similar triangles: can the two triangles be placed one on top of the other such that the corner of one exactly fits with the corner of the other.
Which of these triangles are similar?
Are a and b similar?
Are a and c similar?
Are $a$ and $d$ similar?

We know that $a$ and $d$ are congruent so they MUST also be similar.
Are a and d similar?
How can we use our knowledge of Similar triangles?

If we have a green triangle with sides 4cm and 5 cm as shown
Then a red triangle with a base twice as long will have all its sides twice as long
Lets work out a problem

• The famous detective Sherlock Holmes in the story “The adventure of the Musgrave Ritual” needed to know where the shadow of a tree would be at a certain time of day. However the tree had long since been chopped down.
  
  – No matter thought he if I know the height of the tree and make a shadow of my own I will be able to solve the puzzle.
Where is the shadow?

Sherlock Holmes used a 6 foot staff we will use a 2 m one.
Where is the shadow?

There are two similar triangles here.
Where is the shadow?

The arithmetic is now quite easy.

- A 2m staff casts a 3m shadow that is 1.5 times longer than the staff
- A 22 m tree will cast a shadow that is 22 m x 1.5 = 33m
- The shadow must be 33m from the tree stump
Umbra Recta

• We now know enough about triangles to find out how the Islamic world used them to find:
  – the heights of mountains.
  – the elevation of stars
  – the elevation of the sun (to tell the time)
  – The height of a building and what it would look like before they had finished building it.